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MEASUREMENTS SECTION

THE INCREMENTAL MAGNETIC PROPERTIES OF SILICON-IRON ALLOYS

With particular reference to the Design of Air-Gapped Smoothing Chokes

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SUMMARY

The fundamental considerations involved in defining the incremental magnetic properties of a material are briefly reviewed, and methods of measuring the properties are described. Measured values at 50 c/s and 800 c/s for the various grades of hot-rolled silicon-iron in common use are tabulated. These measurements were obtained on samples representing the best and the worst (as judged by total power loss at 50 c/s and magnetic flux density of 10 000 and 13 000 gauss) for each grade of material, specially selected from production by the manufacturer.

It is shown that the apparent reluctivity of a magnetic circuit, with an air-gap which is optimum for the particular conditions of incremental magnetization, can be related to the apparent polarizing magnetizing force by an empirical equation. This equation is sufficiently accurate for most practical applications, and its use enables a purely analytical method of design of air-gapped smoothing chokes to be evolved. Formulae are developed for designs where the polarizing current and incremental inductance, together with either the d.c. voltage drop or the temperature rise, are specified. Typical values of design parameters used in actual manufactured chokes are given.

For comparison, a small number of measurements of incremental properties of two grades of nickel-iron alloys are included in an Appendix.

(1) LIST OF PRINCIPAL SYMBOLS

H = Magnetizing force, oersteds.
B = Magnetic flux density, gauss.
μ = Relative permeability.
μ_0 = Free-space permeability, gauss/oersted.
$\mu_\Delta/\overline{\theta}$ = Relative complex incremental permeability.
ν = Relative reluctivity.
H'_p = Apparent polarizing magnetizing force in a magnetic circuit containing an air-gap, oersteds.
ν'_Δ = Relative apparent incremental reluctivity of a magnetic circuit containing an air-gap.
$\nu'_{\Delta_{min}}$ = Minimum value of ν'_Δ obtained by appropriate adjustment of gap.
S = Reluctance, gilberts/maxwell.
L = Inductance, henrys.
I = Current, amp.
V = Potential difference, volts.
f = Frequency, c/s.
N = Number of turns in a coil winding.
l = Mean length of flux path in magnetic core, cm.
A = Net cross-sectional area of core calculated from weight and mean specific gravity, cm^2.
x = Ratio of length of air-gap to l.
x_0 = Value of x giving minimum value of ν'_Δ.
l_T = Mean length of turn in coil winding, cm.
P = Power dissipated per unit surface-area of choke-coil winding, watts/cm^2.

Subscript w indicates quantity relating to winding conductor.
Subscript p indicates polarizing (H, B, etc.).
Subscript Δ indicates incremental value (H, B, etc.).
$\hat{}$ indicates peak value of an alternating quantity, or positive peak value where this differs from the negative, e.g. $\hat{H}_\Delta$, $\hat{B}_\Delta$.
$\check{}$ indicates negative peak value of an alternating quantity, e.g. $\check{H}_\Delta$, $\check{B}_\Delta$.
$\bar{}$ indicates r.m.s. value of alternating quantity, e.g. $\bar{V}_\Delta$.

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(2) INTRODUCTION

The purpose of the paper is twofold:

(*a*) To present in a form suitable for design purposes, experimental data on the incremental permeability of various grades of hot-rolled silicon-iron.

(*b*) To describe a purely analytical method of designing air-gapped inductors for use under conditions of combined a.c. and d.c. magnetization, based on empirical formulae derived from the experimental data.

The data were obtained, and the design method developed, primarily for application to the design of the chokes used in smoothing the power supplies in telephone exchanges and repeater stations. In some of the larger stations, individual chokes may weigh as much as a ton, and the cost of the smoothing filter may be comparable with that of the generating equipment. The economic design of the chokes is therefore a matter of importance. The samples of magnetic materials on which the measurements were made were supplied in 1941. The authors have made no measurements to determine whether the tabulated data are typical of current production.

(3) GENERAL THEORY

Fig. 1 shows a typical magnetization curve for a ferromagnetic material, and indicates how the flux density B in an initially unmagnetized specimen varies when the applied magnetizing force is increased smoothly from zero. The corresponding variation of μ ($= B/H \div \mu_0$), the relative permeability, is also shown.*

If, however, the magnetizing force is increased from zero to a value $+ H$; then decreased to zero; increased in the reverse direction to a value $- H$; then decreased to zero again; and the cycle repeated a number of times, the B/H relationship will follow a hysteresis loop of the normal type, symmetrical about the origin. If a number of hyteresis loops are measured, corresponding to different maximum values of H, it is well known that the curve joining the limits or points of these loops is the same as, or for practical purposes sufficiently nearly the same as, the normal magnetization curve of Fig. 1.

Suppose now that a steady or polarizing magnetizing force H_p is applied to an initially unmagnetized specimen, bringing it to the operating point P in Fig. 2. If a slowly alternating magnetizing force (not necessarily sinusoidal) is superimposed on the force H_p, so that the value of H varies smoothly from $H_p - \check{H}_\Delta$ to $H_p + \hat{H}_\Delta$, the B/H relationship will follow the loop SR. It will be observed that the upper limit of the loop, like the limits

* As μ_0, the permeability of free space, is numerically equal to unity in C.G.S. units, it will be omitted throughout the paper, and the adjective "relative" before permeability and its reciprocal, reluctivity, will be dropped.

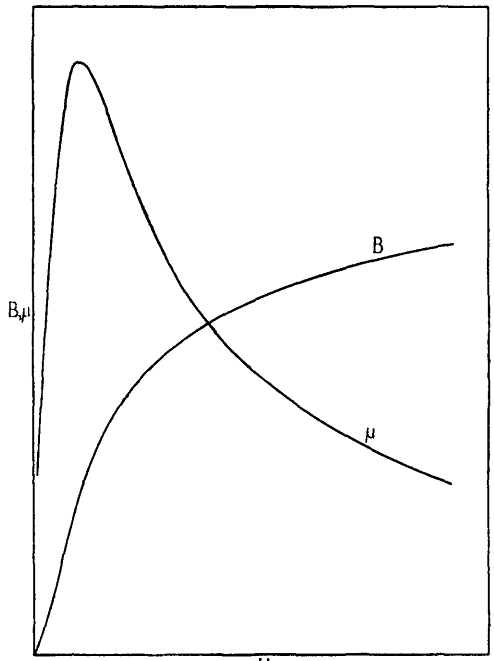

Fig. 1.—Typical magnetization and permeability curves.

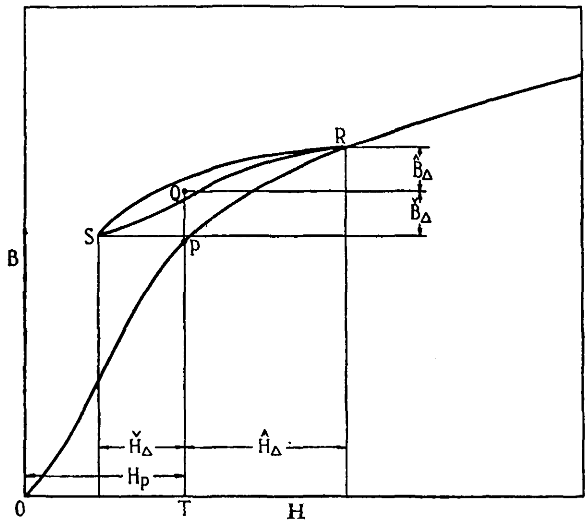

Fig. 2.—Typical hysteresis loop for combined polarizing and alternating magnetization.

of the symmetrical hysteresis loops, lies on the normal magnetization curve. It is also evident that the mean value of the flux density (called the polarizing flux-density) is increased from the value PT to the value QT when the alternating magnetizing force is applied. The ratio QT/OT of the polarizing flux-density B_p to the polarizing magnetizing force H_p will be called the polarization permeability, μ_p. The mean slope of the loop SR will be an approximate measure of the permeability (called the incremental permeability and denoted by μ_Δ) of the material to the superimposed alternating magnetizing force. It is evident that μ_p is greater than μ, the ordinary d.c. permeability for the same value of H, and that μ_Δ is less than either μ or μ_p. Moreover, even if the amplitude of the alternating magnetizing force is reduced to a very small value, the incremental permeability does not approach the slope of the magnetization curve, but is always much less than this.

From the shape of the loop, it is clear that the alternating magnetizing force H_Δ and the alternating flux density B_Δ cannot both be sinusoidal at the same time. When, however, $H_\Delta \ll H_p$ the incremental loop can be assumed to approximate to an ellipse, and if H_Δ is sinusoidal, so also will B_Δ be, though there will be a phase displacement between them, the phase angle depending on the eccentricity of the ellipse.

Under these conditions it is convenient to define the incremental permeability more precisely, as the vector ratio of the alternating component of the flux density to the alternating component of the magnetizing force. When waveform distortion is not negligible, it is recommended by the British Standards Institution that B_Δ should be maintained sinusoidal, and that μ_Δ be taken as the vector ratio of B_Δ to the fundamental component of H_Δ. This condition of sinusoidal flux density holds approximately in many practical applications, and even when this is not so, the resulting error will, in general, be small provided the depth of modulation of H (i.e. $\hat{H}_\Delta/H_p$) is less than 50%. As is evident from Fig. 2, the depth of modulation of B is always less than that of H.

So far, hysteresis effects only have been considered, and it has been assumed that the frequency of the alternating magnetizing force is so low that eddy currents in the magnetic material can be neglected. As the frequency is raised, eddy currents will have an effect on both the magnitude and phase of the resultant flux density corresponding to any particular value of magnetizing force, but it is convenient still to define the incremental permeability as the vector ratio of the resultant flux density to the magnetizing force.

Certain consequences of defining incremental permeability in the above manner will now be discussed. Consider a closed core of uniform cross-section of a ferromagnetic material, and let the polarizing and alternating magnetizing forces be applied by means of a winding carrying direct and alternating currents. Assume for simplicity the ideal condition in which the resistance (including eddy-current effects) in the winding is negligible. Then the alternating potential difference between the ends of the winding will be equal to the induced e.m.f. due to the resultant alternating flux density in the core. If the impedance of the ideal coil is Z, it follows that the value of inductance calculated from the magnitude only of μ_Δ will be equal to the "complex inductance" Z/ω of the coil, where ω is the angular frequency of the alternating magnetizing force. If the coil is considered at any particular frequency as a circuit element having series-connected resistance and inductance, i.e. $Z = R + j\omega L$, and if L is regarded as the "true" inductance at that frequency, it is evident that to calculate L the permeability must be taken not as μ_Δ but as $\mu_\Delta \cos\theta$, where θ is the phase angle of μ_Δ.

At frequencies and thicknesses of laminations for which the effect of eddy currents in the core is not large, the phase angle θ is in general small. As in many applications, it is the impedance rather than the reactance of a coil which is important, and it is often permissible in design to neglect the phase angle of μ_Δ.

(4) RESULTS OF MEASUREMENTS OF INCREMENTAL PROPERTIES OF SILICON-IRON ALLOYS

As the investigation was carried out with the primary object of obtaining the data required in the design of large smoothing-chokes for the power supplies of telecommunication plant, attention was confined to the materials used in their construction, and to the conditions of magnetization normally occurring in this application.

The incremental inductance is required to be within specified limits at an audio frequency generally not above about 1 000 c/s.

The alternating flux density is then usually comparatively low ($\leqslant$ 100 gauss). The fundamental data necessary are (a) μ_p as a function of H_p with superposed alternating flux density $\leqslant$ 100 gauss and (b) μ_Δ as a function of H_p for values of alternating flux density, $\hat{B}_\Delta \leqslant 100$ gauss.

The effect of the magnitude of superposed alternating flux density on the values of μ_p was first investigated. B_p/H_p curves were measured (from which μ_p was calculated) for values of $\hat{B}_\Delta$ of 0, 100, 500 and 1 000 gauss. For convenience the frequency of $\hat{B}_\Delta$ was 50 c/s. Typical results for a ring sample of 4·3% silicon iron (14-mil laminations) are given in Table 1. From these it is clear that, even when $\hat{B}_\Delta$ is as high as 100 gauss, μ_p is not appreciably affected by the presence of alternating flux density, provided that $H_p \geqslant 1$. Values of $H_p < 1$ are of little interest in the design of large smoothing-chokes. The frequency of the alternating flux density has a second-order effect on μ_p and so similar results would be obtained at higher frequencies up to at least 800 c/s.

Table 1

μ_p AS A FUNCTION OF H_p AND $\hat{B}_\Delta$

Ring Sample, 4·3% Silicon-Iron, Thickness 14 mils. Frequency of Alternating Flux Density 50 c/s.

H_p, oersteds	$\hat{B}_\Delta = 0$	$\hat{B}_\Delta = 100$ gauss	$\hat{B}_\Delta = 500$ gauss	$\hat{B}_\Delta = 1\,000$ gauss
0·25	2 400	6 000	—	—
0·50	5 000	7 800	11 600	17 000
1·00	6 800	6 800	8 700	10 000
2·00	4 850	4 850	5 300	5 700
4·0	2 850	2 850	2 970	3 100
8·0	1 560	1 560	1 600	1 630

The relationship between incremental permeability (as a vector quantity $\mu_\Delta/\overline{\theta}$) and H_p was then investigated at 50 c/s and 800 c/s. It was convenient to measure $\mu_\Delta/\overline{\theta}$ for low values of $\hat{B}_\Delta$ at 800 c/s and for high values at 50 c/s. For $\hat{B}_\Delta = 100$ gauss, measurements were made at both frequencies. Table 2 gives typical results for the same sample (4·3% silicon iron) as was used to obtain the results of Table 1. It will be seen that the modulus of the incremental permeability at 800 c/s for $\hat{B}_\Delta = 100$ gauss is about 20% lower than at 50 c/s. The argument is somewhat greater but is of less practical interest.

These preliminary measurements were carried out on a sample having the highest silicon content as it was expected that this would show the greatest variation of μ_p with $\hat{B}_\Delta$ and of μ_Δ with frequency. The results, supplemented by measurements made on other samples (of 4% and 1·5% silicon content) were considered to justify limiting further measurements of μ_p to the condition $\hat{B}_\Delta = 0$ only and of μ_Δ to a frequency of 800 c/s only. Measurements of μ_Δ were further restricted to values of $\hat{B}_\Delta \leqslant 100$ gauss, since, as already mentioned, higher values rarely occur in the class of chokes considered.

Samples in the form of rings and also as strips were supplied by the manufacturer. The samples were taken from two batches of material chosen (on the basis of total loss at 50 c/s and flux density 10 000 and 13 000 gauss) to represent the two extremes of quality for each particular grade of material likely to be met with in production. The total-loss measurements were made by the manufacturer on one half of each sheet, the ring and strip samples being cut from the other half. All samples were from nominally 14- or 20-mil thick sheets. Dimensions were as follows:

Rings, internal diameter 12 cm, external diameter 15 cm.

Strips, length 6 in, width ¾ in.

Details of the grades of material are given in Table 3. The rings were assembled so that each specimen weighed about one kilogramme The cross-sectional area was calculated from the measured weight and the specific gravity. Particulars of the ring specimens when complete with windings were as follows (windings were, of course, applied individually by hand):

Mean length of magnetic path, 42·4, cm.

Cross-sectional area, about 3 cm².

Magnetizing winding, 300 turns (uniformly distributed).

Search winding 160 turns, tapped at 5, 10, 20, 40 and 80.

The strips were assembled in a square in fixed windings with the corners interleaved. The windings were distributed equally along the four sides. Particulars were as follows:

Weight of core, 1 kg, approx.

Mean length of magnetic path, 61 cm.

Cross-sectional area, about 3 cm².

Magnetizing winding, 320 turns.

Search winding, 32 turns, tapped at 4, 8 and 16.

The two types of samples (rings and strips) were measured for two grades of material (1·5% and 4% silicon content) in order to determine to what extent measured values of fundamental data depend upon the form of the specimen. Ring samples are likely to yield more accurate results since the magnetic circuit is continuous and uniform. Strip samples, however, are more convenient since they can be assembled in a permanent set of

Table 2

$\mu_\Delta/\overline{\theta}$ AS A FUNCTION OF H_p AND $\hat{B}_\Delta$

Ring Sample, 4·3% Silicon-Iron, Thickness 14 mils.

Frequency, c/s	800						50							
Alternating flux density, gauss	$\hat{B}_\Delta = 1$		$\hat{B}_\Delta = 10$		$\hat{B}_\Delta = 100$		$\hat{B}_\Delta = 100$		$\hat{B}_\Delta = 200$		$\hat{B}_\Delta = 500$		$\hat{B}_\Delta = 1\,000$	
H_p, oersteds	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$	μ_Δ	$/\overline{\theta}$
0	380	4°	440	6°	980	19°	1 160	11°	1 660	15°	2 030	18°	2 900	23°
0·25	370	4°	430	6°	940	17°	1 100	11°	1 440	14°	1 640	16°	1 980	19°
0·50	350	3°	400	5°	820	15°	900	10°	1 100	13°	1 200	15°	1 450	18°
1·00	270	2°	290	4°	520	12°	570	9°	680	12°	750	14°	930	17°
2·00	160	2°	180	2°	280	8°	300	7°	360	10°	420	12°	530	15°
4·0	90	1°	100	2°	140	6°	150	6°	170	8°	190	9°	260	11°
8·0	60	< 1°	60	< 1°	70	3°	70	4°	80	5°	90	6°	120	7°

Table 3

DETAILS OF MATERIALS MEASURED

Grade of material	Approximate silicon content	Specific gravity	Approximate thickness, mils	Total loss at 50 c/s, watts/lb		Type of sample tested
				(Flux density 10 000 gauss)	(Flux density 13 000 gauss)	
Lohys	0·2%	7·82	18	1·20 1·43	— —	Rings
Medium-resistance	1·5%	7·75	20	0·93 0·96	— —	Rings and strips
41 quality	3·0%	7·60	14	0·71 0·75	1·25 1·31	Rings
Stalloy	4·0%	7·55	14	0·60 0·62	1·00 1·06	Rings and strips
Super-Stalloy	4·3%	7·55	12	0·47	0·84	Rings

Where two values of total loss are given, these are averages of actual measured values of sheets from batches chosen to represent the two extremes of quality for the grade likely to arise in production.

fixed windings; ring samples have, of course, to be wound individually. The use of strips also enables the properties of the material along and at right angles to the direction in which the sheet was rolled to be measured separately. In the construction of a large choke, the core is usually assembled from rectangular strips so that it is possible for these to be cut in a particular direction if any advantage is gained thereby.

The method of assembling the strip samples has some effect on the measured fundamental data. The results given in Table 4

Table 4

EFFECT ON MEASURED PERMEABILITIES OF ASSEMBLING STRIP SAMPLES WITH BUTT JOINTS

4% Silicon Iron, Thickness 14 mils. Frequency of Alternating Flux Density 800 c/s.

H_p, oersteds	$\hat{B}_\Delta = 0$ μ_p ratio	$\hat{B}_\Delta = 10$		$\hat{B}_\Delta = 100$	
		μ_Δ ratio	$/\theta$ difference	μ_Δ ratio	$/\theta$ difference
0	—	0·68	− 2°	0·50	− 11°
0·25	0·23	0·68	− 2°	0·52	− 9°
0·5	0·16	0·74	− 1°	0·65	− 7°
1·0	0·15	0·99	0°	0·75	− 4°
2·0	0·23	1·32	1°	1·15	− 1°
4·0	0·33	1·68	1°	1·52	1°
8·0	0·60	2·28	2°	2·09	2°

μ_p ratio denotes the ratio of μ_p with butt joints to μ_p with the corners interleaved. μ_Δ ratio denotes the corresponding ratio of μ_Δ.

$/\theta$ difference denotes $/\theta$ with butt joints minus $/\theta$ with the corners interleaved.

show the effect (with a sample of 4% silicon-iron strips) of using a butt joint at the corners instead of interleaving.*

For the particular application of designing gapped chokes, the error due to working from fundamental data obtained with butt-jointed cores would, however, be very small.† This is because

* The values of H_p given in Table 4 are those calculated from the magnetizing ampere-turns and the dimensions of the core. Since a butt joint and, to a lesser extent, an interleaved joint are equivalent to a small air-gap, the actual values of H_p in the iron will be smaller, though they cannot be determined without knowing the precise values of the equivalent air-gaps. This is the chief cause of the difference in the results for interleaved and butt joints. The effect is very marked and demonstrates that the corners must be interleaved if results approximating closely to the correct fundamental data are to be obtained.

† If the inductance of any choke with optimum air-gap is calculated from data obtained (a) from ring, (b) from interleaved and (c) from butt-jointed samples respectively, the discrepancy between (a) and (b) has been found not to exceed 6% and between (a) and (c) not to exceed 10%, although the effect of the form of the sample on μ_p and μ_Δ is, in general, very much greater than this, as illustrated in Tables 4 and 5.

Table 5

EFFECT ON MEASURED PERMEABILITIES OF USING STRIP SAMPLES ASSEMBLED IN DIFFERENT WAYS

1·5% Silicon-Iron, Thickness 20 mils.

Arrangement of strips (see below)	H_p, oersteds	μ_p ratio ($\hat{B}_\Delta = 0$)	μ_Δ ratio ($\hat{B}_\Delta = 1$ gauss)	μ_Δ ratio ($\hat{B}_\Delta = 10$ gauss)	μ_Δ ratio ($\hat{B}_\Delta = 100$ gauss)
A	0	—	0·87	0·88	0·80
	0·25	1·0	0·87	0·88	0·86
	0·5	1·0	0·90	0·89	0·87
	1·0	1·0	0·95	0·93	0·87
	2·0	0·87	1·03	0·98	0·89
	4·0	0·86	1·16	1·05	0·96
	8·0	0·89	1·35	1·15	1·12
B	0	—	1·01	1·00	0·96
	0·25	1·0	0·99	0·97	1·01
	0·5	1·0	1·00	0·98	1·02
	1·0	0·97	1·05	1·01	1·03
	2·0	0·87	1·13	1·07	1·06
	4·0	0·84	1·24	1·17	1·15
	8·0	0·86	1·37	1·29	1·34
C	0	—	1·06	0·96	0·92
	0·25	1·0	1·02	0·95	0·97
	0·5	1·0	1·06	0·97	0·98
	1·0	1·0	1·13	1·02	0·98
	2·0	0·85	1·23	1·09	1·01
	4·0	0·88	1·32	1·19	1·13
	8·0	0·89	1·37	1·32	1·35

1. μ_p ratio denotes the ratio of μ_p for the strip sample to μ_p for the ring sample. μ_Δ ratio denotes the corresponding ratio of μ_Δ.

2. The same set of strips were used in each arrangement and were cut from the same sheets as the rings. The strips were a mixture in equal proportions of those cut longitudinally and transversely with respect to the direction of rolling.

Arrangements A, B and C were as follows:—

A. All layers consisted of two transverse strips and two longitudinal strips, opposite strips being similar. All layers were identical, i.e. each side of the square contained either all transverse or all longitudinal strips. Layers were interleaved singly at the corners.

B. As A except that transverse and longitudinal strips were interchanged in alternate layers.

C. As B except that layers were interleaved in pairs at the corners.

3. Within the limits of accuracy of the measurements, the values of $/\theta$ were independent of the arrangement and were equal to those for the rings.

the design problem amounts to answering the following question: "What is the optimum air-gap to be inserted in this core for the given conditions of magnetization, and what is then the performance of the core?" It is evident that the same answer should be obtained to the second part of the question, irrespective of whether the core on which the fundamental data were obtained had already a small gap or not. There will be a small effect on the value of optimum air-gap, but, as final setting of gaps is normally carried out experimentally, this is of negligible importance. Nevertheless strip samples were always tested with the corners interleaved. The data then approximate as closely as possible to the true properties of the material, and should therefore be more generally useful.

The effect of assembling strip samples by different methods is shown in Table 5 for the case of 1·5% silicon iron. The results for the different methods of assembly are similar, though they differ somewhat from those obtained with ring samples. It was decided that where longitudinal and transverse strips were mixed, assembly method B should be used as this was considered to correspond most nearly to the conditions with the ring specimens. (Rings were assembled with random relative orientation of the directions of rolling.)

Measurements on the complete set of samples as given in Table 3 were carried out as follows. B_p was measured as a function of H_p with no superposed alternating flux density (corresponding to use when $\hat{B}_\Delta \leqslant 100$ gauss). $\mu_\Delta/\bar{\theta}$ was measured as a function of H_p with superposed alternating flux density $\hat{B}_\Delta$ of 1, 10 and 100 gauss, at a frequency of 800 c/s. The results of these measurements are given in Tables 6, 7, 8, 9, 10, 11 and 12.

It will be seen that increase in approximate silicon-content is accompanied by increase in both μ_p and μ_Δ. The increase in silicon content is also accompanied by a decrease in the loss per pound. However, differences in loss per pound between the samples of nominally the same material are not always reflected in this way in the values of μ_p and μ_Δ. It may be pointed out the loss values are not constant in each batch and in part the dispersion is comparable with the difference between the average losses per pound of the two samples of nominally the same material.

The effect on μ_p and μ_Δ of cutting the strip samples in a longitudinal or transverse direction with respect to the direction of rolling was investigated by comparing results for cores, composed of strips of 1·5% and 4% silicon iron cut in each of these two ways, with corresponding results for cores, composed of a mixture of longitudinally and transversely cut strips. The values of μ_p were on the average 20% higher for longitudinal than for transverse strips. μ_Δ was usually higher for the longitudinal than the transverse but never more than about 10% higher, and for one sample it was actually lower. The combined effect of these differences in μ_p and μ_Δ on the incremental inductance of a smoothing choke is that a choke of material magnetized longitudinally is often 10% greater in inductance than one using transverse magnetization. The improvement varies with the material and with $\hat{B}_\Delta$ and, indeed, in some cases there is no improvement.

(5) THEORY AND DESIGN OF AIR-GAPPED INDUCTORS UNDER CONDITIONS OF COMBINED D.C. AND A.C. MAGNETIZATION

(5.1) Theory of the Air-Gapped Magnetic Circuit

The commonest practical application of the data on incremental permeability is to the design of chokes or inductance coils carrying both alternating and direct currents and having a magnetic circuit containing an air-gap.

In considering the principles of design, the following simplifying assumptions will be made:

(*a*) The core is of uniform cross-section.

(*b*) The gap length is small compared with the cross-sectional dimensions of the core, so that very little fringing of the flux occurs at the edges of the gap.

(*c*) Flux leakage is negligible, so that at any instant the flux in all parts of the core is the same, and is equal to the flux in the gap. In addition, all the flux links every turn of the winding.

These assumptions, particularly (*b*), do not hold accurately in practice, but the general conclusions are not greatly affected. For example, the actual air-gap length required to obtain certain required electrical characteristics in a coil will differ slightly

Table 6

μ_p AS A FUNCTION OF H_p

Ring samples. $\hat{B}_\Delta = 0$

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersted									
0·25	800	400	800	800	1 600	1 600	2 000	2 000	3 600
0·5	1 000	600	1 400	1 400	2 600	2 600	3 000	3 000	6 200
0·75	1 600	940	2 400	2 400	4 000	4 540	4 000	4 000	6 540
1·0	2 500	1 700	3 500	3 200	4 300	4 700	4 500	4 100	6 200
1·25	3 040	2 400	3 920	3 520	4 240	4 640	4 400	4 080	5 680
1·5	3 470	2 800	4 000	3 600	4 070	4 400	4 200	3 940	5 270
1·75	3 660	3 150	3 890	3 550	3 890	4 120	3 950	3 720	4 860
2·0	3 600	3 250	3 750	3 450	3 650	3 850	3 750	3 500	4 450
2·25	3 520	3 290	3 600	3 340	3 470	3 650	3 520	3 290	4 180
2·5	3 400	3 240	3 440	3 200	3 280	3 440	3 320	3 120	3 920
3·0	3 140	3 040	3 170	2 940	3 000	3 100	2 970	2 840	3 440
3·5	2 860	2 780	2 890	2 690	2 720	2 830	2 720	2 600	3 060
4·0	2 630	2 550	2 630	2 480	2 500	2 600	2 480	2 380	2 780
5·0	2 260	2 220	2 220	2 140	2 160	2 220	2 140	2 060	2 340
6·0	2 000	1 980	1 940	1 890	1 890	1 940	1 870	1 800	2 020
8·0	1 600	1 590	1 540	1 520	1 520	1 550	1 490	1 450	1 580
10·0	1 340	1 320	1 280	1 260	1 260	1 290	1 250	1 220	1 300

Table 7

μ_Δ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta = 1$ gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	250	230	220	250	260	250	370	430	470
0·25	250	230	220	250	260	250	370	420	450
0·5	250	230	220	240	250	250	370	400	410
0·75	240	230	210	240	230	240	350	360	370
1·0	230	220	200	220	220	230	320	320	320
1·25	220	210	180	200	200	210	290	290	290
1·5	210	200	170	190	190	200	260	260	250
1·75	200	190	150	170	170	180	240	240	230
2·0	180	180	140	160	160	170	230	230	210
2·25	170	160	130	150	150	160	210	210	190
2·5	160	150	130	140	140	150	200	200	180
3·0	150	140	120	130	130	140	180	180	170
3·5	130	120	110	120	120	120	170	170	150
4·0	120	110	100	110	120	120	150	150	140
5·0	100	100	80	90	100	100	130	130	110
6·0	90	80	70	80	90	90	110	110	100
8·0	70	70	60	80	70	70	90	90	80
10·0	60	60	50	60	60	60	60	60	70

Table 8

$/\bar{\theta}$ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta = 1$ gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mil	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	8·5°	7·5°	5·5°	6·0°	2·5°	2·0°	6·0°	6·0°	5·5°
0·25	8·5°	7·0°	5·5°	6·0°	2·0°	1·5°	6·0°	6·0°	5·5°
0·5	8·5°	7·0°	5·0°	5·5°	2·0°	1·5°	5·5°	5·5°	5·0°
1·0	8·0°	6·5°	4·5°	5·0°	1·5°	1·5°	5·0°	5·0°	3·5°
2·0	5·5°	5·0°	3·5°	4·0°	1·5°	1·5°	4·0°	4·0°	2·5°
4·0	4·0°	3·0°	2·0°	2·5°	1·5°	1·5°	3·5°	3·5°	2·5°
8·0	2·5°	2·0°	0·5°	1·0°	1·0°	1·0°	2·5°	2·5°	2·5°

from the theoretical gap length, but the dimensions of the core and number of turns will not, in general, require appreciable modification.

Design formulae will be developed in terms of practical C.G.S. electromagnetic units.

If the cross-sectional area of the core in cm² is A, the flux density in gauss is B and the winding has N turns carrying a current of I amp, then

$$BA = \frac{0{\cdot}4\pi NI}{S} \qquad (1)$$

where S is the total reluctance of the magnetic circuit in gilberts/maxwell. If the mean length of flux path in the core is l and the length of air-gap is xl, both in cm, then

$$S = \frac{l}{A}\left(\frac{1}{\mu} + x\right)$$

where μ is the permeability of the core, appropriate to the condition of magnetization.

The quantity x will be called the gap ratio.

From eqn. (1),
$$B = \frac{0{\cdot}4\pi NI}{l(1/\mu + x)} \qquad (2)$$

When the current contains an alternating component I_Δ superposed on a polarizing component I_p the equation may be applied separately to the polarizing and incremental quantities, provided that the appropriate values of μ are used, i.e.

Table 9

μ_Δ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta$ = 10 gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	280	260	260	290	310	320	570	510	600
0·25	280	260	260	290	300	310	560	500	590
0·5	280	260	250	280	290	300	520	490	560
0·75	280	260	240	270	270	280	470	420	510
1·0	260	240	220	250	250	260	420	370	460
1·25	250	220	200	230	230	240	380	330	410
1·5	230	200	190	210	210	220	350	300	380
1·75	210	190	170	190	200	200	320	280	340
2·0	190	180	160	180	190	190	300	260	310
2·25	180	160	150	160	180	180	280	240	280
2·5	160	150	140	150	170	170	270	230	260
3·0	150	140	130	140	150	150	250	210	220
3·5	130	120	120	130	140	140	230	200	190
4·0	120	110	110	120	130	130	210	190	170
5·0	100	100	90	100	110	110	180	170	140
6·0	90	80	80	90	100	100	160	150	120
8·0	70	70	80	80	80	80	130	110	90
10·0	60	60	60	60	70	70	110	100	80

Table 10

$\angle\theta$ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta$ = 10 gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	10·0°	9·0°	9·0°	9·0°	4·0°	4·5°	8·5°	11·0°	14·0°
0·25	10·0°	9·0°	9·0°	9·0°	3·5°	4·0°	7·5°	10·0°	12·5°
0·5	9·5°	8·5°	8·0°	8·0°	3·0°	3·5°	7·0°	9·0°	11·0°
1·0	9·0°	8·5°	7·0°	7·0°	3·0°	3·5°	6·5°	8·0°	9·0°
2·0	6·5°	6·0°	5·5°	5·5°	2·5°	3·5°	5·5°	6·5°	7·0°
4·0	4·5°	4·0°	3·5°	3·5°	2·5°	3·0°	5·0°	6·0°	5·5°
8·0	3·5°	2·5°	2·0°	2·0°	2·0°	3·0°	3·5°	4·0°	3·5°

$$B_p = \frac{0 \cdot 4\pi N I_p}{l(1/\mu_p + x)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\hat{B}_\Delta = \frac{0 \cdot 4\pi N \hat{I}_\Delta}{l(1/\mu_\Delta + x)} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The quantity $\frac{0 \cdot 4\pi N I_p}{l}$ has the dimensions of magnetizing force and will be denoted by H'_p. It is the value which the magnetizing force would have if the magnetic circuit were homogeneous and of length l equal to that of the core without air-gap.

From eqn. (3),

$$H'_p = B_p\left(\frac{1}{\mu_p} + x\right) = H_p(1 + \mu_p x)$$

where H_p is the true value of the polarizing magnetizing force in the core.

Hence
$$x = \frac{1}{\mu_p}\left(\frac{H'_p}{H_p} - 1\right) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For any given value of $\hat{B}_\Delta$, the variation of μ_p with H_p must be experimentally determined (typical values for 4·3% silicon iron are given in Table 1) and, for the material in question, a series of curves can then be derived of x against H_p for various values of H'_p. Fig. 4 shows a typical set of such curves for 4% silicon iron at low values of $\hat{B}_\Delta$ (less than about 100 gauss), when the magnetization curve is not much affected by the precise value of $\hat{B}_\Delta$. Curves for higher values of $\hat{B}_\Delta$ would, however, be of similar shape.

Table 11

μ_Δ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta = 100$ gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	460	400	510	530	660	690	1 050	970	1 080
0·25	450	390	480	500	620	650	1 000	940	1 030
0·5	430	370	430	450	560	600	930	830	950
0·75	410	350	380	400	500	520	780	710	810
1·0	380	330	350	370	450	450	680	600	730
1·25	350	310	320	340	400	400	590	510	650
1·5	310	280	290	310	370	370	530	470	590
1·75	280	260	270	280	340	340	470	430	550
2·0	260	240	250	260	320	310	440	400	510
2·25	230	220	230	240	300	290	410	380	470
2·5	220	200	220	220	280	270	390	350	440
3·0	190	180	190	190	250	240	350	320	390
3·5	170	160	170	170	230	210	320	290	340
4·0	150	140	150	150	200	190	290	270	310
5·0	130	120	130	130	170	150	240	230	250
6·0	110	110	120	120	140	130	210	200	210
8·0	90	90	90	90	110	100	160	150	150
10·0	70	70	70	70	90	90	130	130	120

Table 12

$/\overline{\theta}$ AS A FUNCTION OF H_p

Ring Samples. $\hat{B}_\Delta = 100$ gauss, frequency 800 c/s

Material	Lohys		Medium-resistance		41 quality		Stalloy		Super-Stalloy
Thickness, mils	18		20		14		14		12
Loss at 50 c/s, watts/lb (Flux density 10 000 gauss)	1·20	1·43	0·93	0·96	0·71	0·75	0·60	0·62	0·47
H_p, oersteds									
0	25·5°	22·0°	26·5°	25·0°	14·0°	14·5°	13·5°	22·0°	22·0°
0·25	24·5°	21·0°	24·0°	22·5°	12·5°	13·0°	12·5°	19·0°	20·0°
0·5	23·0°	20·0°	21·5°	20·0°	11·0°	11·0°	11·0°	17·0°	17·5°
1·0	19·5°	17·0°	18·0°	16·5°	9·0°	9·0°	9·5°	14·0°	14·0°
2·0	14·0°	12·5°	13·5°	12·5°	8·5°	8·0°	8·0°	11·5°	11·0°
4·0	10·5°	9·5°	10·5°	9·5°	7·5°	7·0°	7·0°	9·5°	8·5°
8·0	8·5°	7·5°	8·0°	7·5°	6·0°	5·5°	5·0°	6·5°	6·0°

If the inductance to alternating current (i.e. the incremental inductance) in henrys is denoted by L_Δ, then, from elementary theory,

$$L_\Delta = \frac{\hat{B}_\Delta AN}{\hat{I}_\Delta} \times 10^{-8} \text{ henrys}$$

By substituting the value of $\hat{B}_\Delta$ from eqn. (4),

$$L_\Delta = \frac{0 \cdot 4\pi N^2 A}{l(1/\mu_\Delta + x)} \times 10^{-8} \text{ henrys} \quad . \quad . \quad . \quad (6)$$

The incremental reluctivity ν_Δ of the core material is equal to $1/\mu_\Delta$. The quantity $(1/\mu_\Delta + x)$ is the reluctivity of a hypothetical homogeneous material having the dimensions of the actual core without gap and an incremental reluctance equal to that of the whole magnetic circuit. It will therefore be called the apparent incremental reluctivity, denoted by ν'_Δ.

$$\text{Hence} \qquad L_\Delta = \frac{0 \cdot 4\pi N^2 A}{l\nu'_\Delta} \times 10^{-8} \text{ henrys} \quad . \quad . \quad . \quad (7)$$

Since μ_Δ is a complex quantity, having magnitude and phase angle, ν'_Δ is complex and L_Δ, as in the simple case with no air-gap, is also complex. If the incremental permeability is $\mu_\Delta/\overline{\theta}$, then

$$\nu'_\Delta = \frac{1}{\mu_\Delta}(\cos\theta + \mu_\Delta x + j\sin\theta) \quad . \quad . \quad . \quad (8)$$

and eqn. (6) becomes

$$L_\Delta = \frac{0 \cdot 4\pi N^2 A \mu_\Delta}{l} \times 10^{-8}\left(\frac{\cos\theta + \mu_\Delta x}{1 + 2\mu_\Delta x \cos\theta + \mu_\Delta{}^2 x^2} - j\frac{\sin\theta}{1 + 2\mu_\Delta x \cos\theta + \mu_\Delta{}^2 x^2}\right)$$

$$= \frac{0 \cdot 4\pi N^2 A \mu_\Delta \times 10^{-8}}{l(1 + 2\mu_\Delta x \cos\theta + \mu_\Delta{}^2 x^2)} \Big/ \arctan \frac{\sin\theta}{\cos\theta + \mu_\Delta x} \text{ henrys} \quad . \quad (9)$$

For the materials, frequencies, and range of flux densities considered, θ is always fairly small ($< 30°$) and the angle of L_Δ will always be smaller than θ. For most applications it is the magnitude of L_Δ which is the greatest interest, the phase angle being unimportant, and with these values of θ the error in the magnitude of L_Δ due to neglecting θ completely is very small. In the further analysis of the air-gapped magnetic circuit the phase angle of μ_Δ will therefore be neglected.

From Figs. 3 and 4, families of curves of ν'_Δ against x for various values of H_p can be derived, a separate family being obtained for each value of $\hat{B}_\Delta$. Fig. 5 shows a set of curves for 4% silicon iron at values of $\hat{B}_\Delta$ below about 10 gauss, when μ_Δ and therefore ν'_Δ are little affected by the precise value of $\hat{B}_\Delta$.

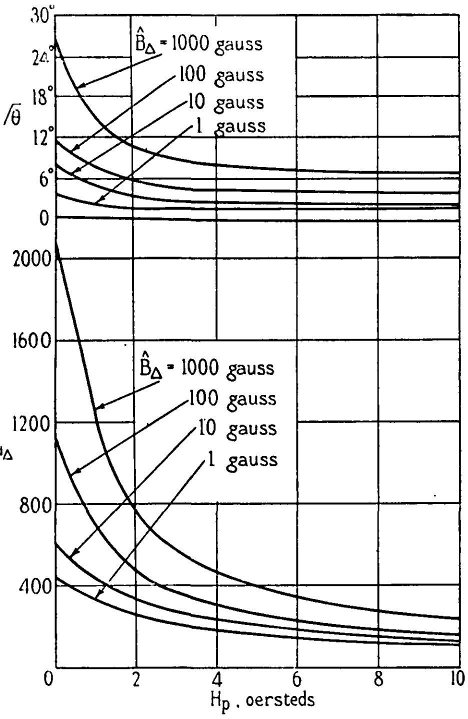

Fig. 3.—μ_Δ and $/\theta$ as functions of H_p and $\hat{B}_\Delta$.

(5.2) Effect of Varying the Air-Gap of a Given Coil

It is assumed that the polarizing current and the magnitude and frequency of the alternating voltage applied to the coil remain constant as the gap is varied. It is further assumed that the a.c. resistance of the winding is small compared to the a.c. impedance under the operating conditions, an assumption which is almost always justified in practice. The applied alternating voltage V_Δ is then equal to the r.m.s. value of the e.m.f.

Hence $\quad \hat{V}_\Delta = \sqrt{(2)}\pi \hat{B}_\Delta A N f \times 10^{-8}$ volts

where f is the frequency of the applied alternating voltage,

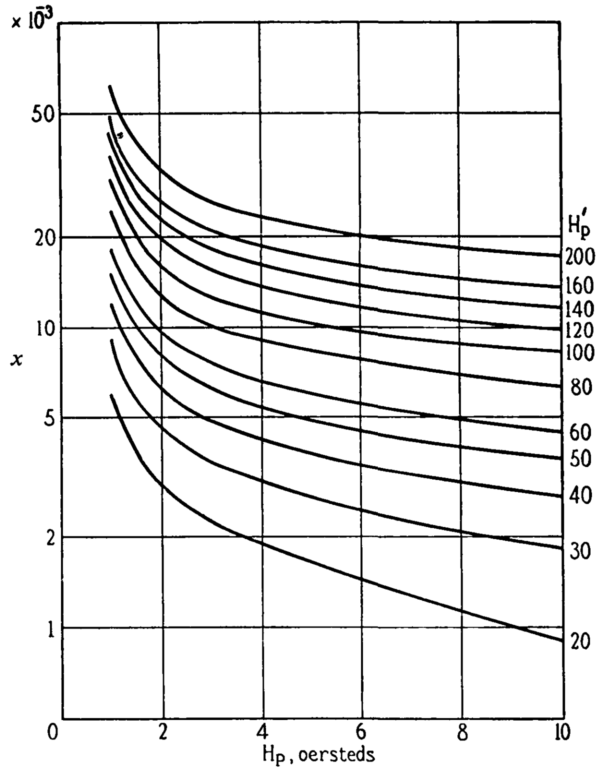

Fig. 4.—Air-gap ratio x as a function of H_p and H'_p.

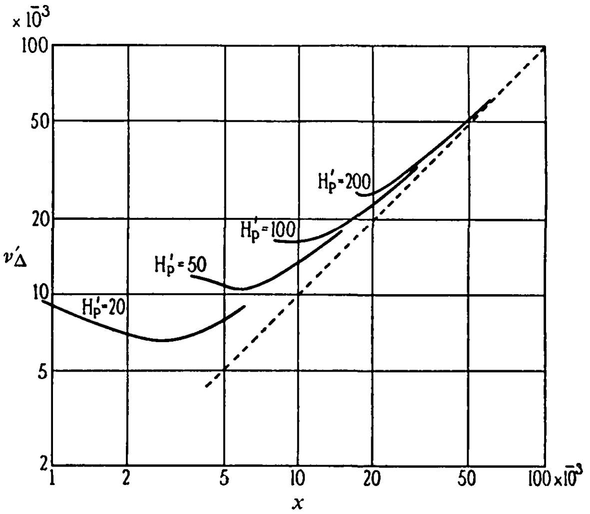

Fig. 5.—ν'_Δ as a function of air-gap ratio x and H'_p.

from which $\quad \hat{B}_\Delta = \dfrac{\hat{V}_\Delta \times 10^8}{\sqrt{(2)}\pi A N f}$ gauss $\quad . \quad . \quad . \quad . \quad (10)$

Since H'_p is proportional to I_p, it follows that, for the conditions specified, $\hat{B}_\Delta$ and H'_p remain constant as the gap is varied, and the variation of ν'_Δ with gap is given by the appropriate curve of the type shown in Fig. 5.

It will be seen that the length of gap giving minimum apparent incremental reluctivity and therefore maximum inductance varies with the value of H'_p, i.e., for any particular coil, with the value of I_p. From curves of the type shown in Figs. 4 and 5, and similar curves for other values of $\hat{B}_\Delta$, curves can be derived showing the minimum value of the apparent incremental reluctivity (denoted by $\nu'_{\Delta_{min}}$) against H'_p, a separate curve being plotted for each value of $\hat{B}_\Delta$. Other curves giving x_0 the corresponding value of the gap ratio can also be plotted. Typical curves for 4% silicon steel are shown in Fig. 6.

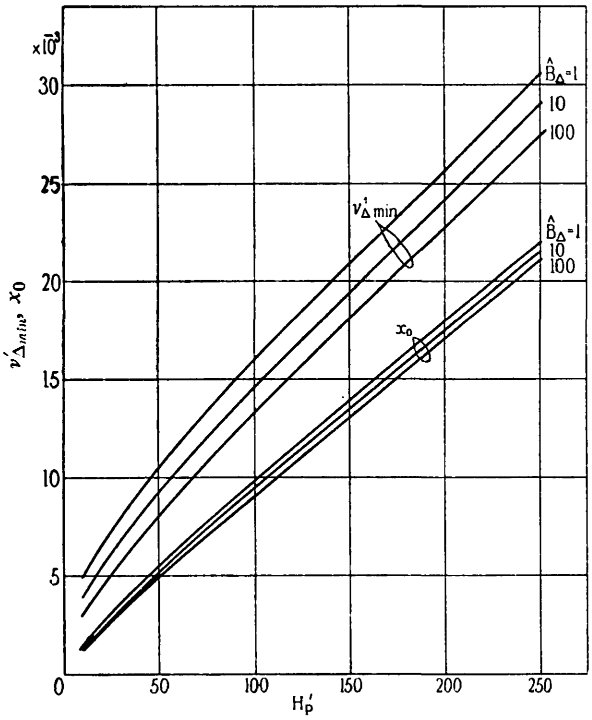

Fig. 6.—$\nu'_{\Delta_{min}}$ and x_0 as functions of H'_p.

the design work could become direct and entirely analytical. It was found that for any particular value of $\hat{B}_\Delta$, the curve connecting the logarithms of $\nu'_{\Delta_{min}}$ and H'_p approximates, for any of the magnetic materials studied, to a straight line. It follows that we can write

$$\nu'_{\Delta_{min}} = \alpha(H'_p)^\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where α and β are constants to be determined experimentally for the particular material and value of $\hat{B}_\Delta$. The corresponding optimum gap ratio can be expressed by a similar empirical equation

$$x_0 = \alpha_1(H'_p)^{\beta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

though α_1 and β_1 are of less practical importance since air-gaps are usually adjusted experimentally.

Table 13 gives the values of α, β, α_1 and β_1 derived from the experimental results given in Tables 6 to 12, $\nu'_{\Delta_{min}}$ and H'_p being measured in C.G.S. electromagnetic units, i.e. in oersteds/gauss and oersteds respectively.

(5.4) Analytical Design Method, Based on the Empirical Expression for $\nu'_{\Delta min}$

It may be assumed that whatever the size and shape of core and number of turns, the gap will be adjusted to give the lowest possible value of ν'_Δ under the specified conditions of polarizing and incremental magnetization. Eqn. (7) may therefore be written

$$L_\Delta = \frac{0 \cdot 4\pi N^2 A}{l\nu'_{\Delta_{min}}} \times 10^{-8} \text{ henrys} \quad . \quad . \quad . \quad (13)$$

Table 13

SUMMARY OF AVERAGE VALUES FOR α, β, α_1 AND β_1

Ring Samples. Frequency of Alternating Flux Density 800 c/s

Material (thickness, mils)	Minimum apparent incremental reluctivity $\nu'_{\Delta_{min}}$						Optimum air-gap ratio x_0	
	α			β			α_1	β_1
	$\hat{B}_\Delta = 1$ gauss	$\hat{B}_\Delta = 10$ gauss	$\hat{B}_\Delta = 100$ gauss	$\hat{B}_\Delta = 1$ gauss	$\hat{B}_\Delta = 10$ gauss	$\hat{B}_\Delta = 100$ gauss	$\hat{B}_\Delta = 1, 10$ or 100 gauss	$\hat{B}_\Delta = 1, 10$ or 100 gauss
Lohys (18)	0·0015	0·0014	0·0012	0·55	0·56	0·56	0·00017	0·89
Medium-resistance (20) . .	0·0021	0·0016	0·0011	0·49	0·53	0·59	0·00028	0·79
41 quality (14)	0·0016	0·0015	0·00079	0·53	0·54	0·64	0·00023	0·82
Stalloy (14)	0·0011	0·00090	0·00060	0·59	0·62	0·68	0·00017	0·88
Super-Stalloy (12) . .	0·00088	0·00064	0·00045	0·63	0·68	0·73	0·00020	0·84

(5.3) Determination of Optimum Design of Coil to Meet Specified Performance Requirements

The method of the preceding Section is similar to others described elsewhere, and enables the performance of a given core wound with a given number of turns to be readily determined for any particular condition of polarization and applied alternating voltage.

The more usual practical design problem, however, is to determine the design of a coil of minimum size to give a specified inductance under particular conditions of polarization and applied a.c. voltage, and with a given maximum d.c. resistance, or more rarely, a given maximum temperature rise. Methods previously described involve graphical constructions or trial-and-error methods; it was considered that if a sufficiently accurate empirical expression could be found for the curves of $\nu'_{\Delta_{min}}$,

By definition $$H'_p = \frac{0 \cdot 4\pi N I_p}{l} \text{ oersteds} \quad . \quad . \quad . \quad . \quad (14)$$

(5.4.1) Design for a Given Maximum D.C. Voltage Drop.

If V_p is the permissible d.c. voltage drop in the coil when carrying a current I_p amp

$$V_p = I_p R_w = \rho_w \frac{l_T N I_p}{A_w} \quad . \quad . \quad . \quad . \quad (15)$$

where R_w is the d.c. resistance of the winding in ohms,

ρ_w is the resistivity of the conductor, ohm-cm,
l_T is the mean length of turn, cm,
A_w is the cross-sectional area of the conductor, cm^2.

The following substitutions will now be made:

$$k_1 = \frac{A_w N}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$k_2 = \frac{l_T}{\sqrt{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$\chi = \frac{\sqrt{A}}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$A_w N$ is the total conductor cross-section and l^2 is a measure of the size of the core. Hence k_1 might be called the winding-space efficiency-factor, interpreted broadly to include not merely the extent to which the available winding space is utilized, but also the efficiency of the core shape in providing winding space. The maximum possible value of k_1 is $\frac{1}{4}\pi$ and would be given by a circular core of negligible radial width, with the circular window completely filled with winding conductors having insulation of negligible thickness. Since such conditions are impracticable, k_1 will normally be very much less than this. The winding "looseness factor" is k_2, and the minimum possible value for this is $2\sqrt{\pi}$, given by a core of circular cross-section and a winding of negligible depth. With reasonable practical shapes of core and winding, k_1 and k_2 will not vary much, though k_1 will tend to be smaller (and possibly k_2 slightly larger) for chokes in which, to assist cooling, the winding fills only a small fraction of the available space.

χ is called the core-shape factor; it is large for short cores of large cross-section and small for long thin cores. The determination of the best value of χ is part of the design problem. Typical practical values of k_1, k_2 and χ are given in Table 14.

From eqns. (13) and (20),

$$H'_p = 0 \cdot 4\pi\left(\frac{V_p I_p}{lK\chi}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and

$$N = \left(\frac{V_p l}{I_p K\chi}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

When H'_p has been determined, the value of x_0, the optimum gap ratio, can be calculated from eqn. (12).

As already stated, K is approximately a constant for all chokes, provided the same conductor material (in practice invariably copper) is used for the winding; α and β are experimentally determined constants for the particular core material. It remains to determine the optimum value of χ, the core-shape factor.

χ may be chosen in order to obtain a choke design for either (*a*) minimum volume, (*b*) minimum weight, or (*c*) minimum cost. The last condition will not be considered here, since the total cost is not dependent solely on the cost of raw materials, but includes manufacturing cost which will vary somewhat with size and form of construction.

Considering first condition (*a*), it is evident from eqn. (23) that the volume of the core of a choke is proportional to $\chi^{\frac{2-\beta}{4+\beta}}$.

The volume of the winding conductor

$$= NA_w l_T = k_1 k_2 l^2 \sqrt{A}$$

$$= \frac{k_1 k_2}{\chi} \times lA = \frac{k_1 k_2}{\chi} \times \text{(core volume)}$$

Table 14

PARTICULARS OF TYPICAL SMOOTHING CHOKES

Current rating I_p, amp	Inductance L_Δ, millihenrys	Voltage drop V_p, volts	Approximate weight, lb	H'_p, oersteds	$k_1 = \frac{A_w N}{l^2}$	$k_2 = \frac{l_T}{\sqrt{A}}$	$\chi = \frac{\sqrt{A}}{l}$
1	700	5·4	11	35	0·0064	6·5	0·14
10	40	1·9	33	73	0·0123	6·4	0·11
200	1·3	0·45	260	110	0·0085	5·5	0·08
500	0·72	0·40	730	130	0·0082	5·4	0·11
1 500	0·079	0·17	780	130	0·0076	5·3	0·11
2 200	0·094	0·30	1 460	150	0·0046	8·6	0·07

In order to simplify subsequent equations, the following additional substitution is made:

$$K = \rho_w \frac{k_2}{k_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Substituting (16), (17), (18) and (19) in (13) and (15), we have

$$L_\Delta = \frac{0 \cdot 4\pi N^2 \chi^2 l}{\nu'_{\Delta_{min}}} \times 10^{-8} \text{ henrys} \quad . \quad . \quad . \quad (20)$$

and

$$V_p = \frac{I_p K N^2 \chi}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

From eqns. (13), (19), (20), and the empirical eqn. (11), the unknown quantities N, $\nu'_{\Delta_{min}}$, and H'_p can be eliminated and the following solution found for l, the mean length of flux path in the core:

$$l = \left[\frac{\alpha^2 L_\Delta^2 K^{2-\beta} I_p^{2+\beta} \times 10^{16}}{(0 \cdot 4\pi)^{2-2\beta} \chi^{2+\beta} V_p^{2-\beta}}\right]^{\frac{1}{4+\beta}} \quad . \quad . \quad (22)$$

Hence the volume of the core is

$$Al = \chi^2 l^3 = \left[\frac{\chi^{2-\beta} K^{3(2-\beta)} \alpha^6 L_\Delta^6 I_p^{3(2+\beta)} \times 10^{48}}{(0 \cdot 4\pi)^{6(1-\beta)} V_p^{3(2-\beta)}}\right]^{\frac{1}{4+\beta}} . \quad (23)$$

Hence the total volume of core plus winding conductor is proportional to

$$\left(\frac{k_1 k_2}{\chi} + 1\right)\chi^{\frac{2-\beta}{4+\beta}}$$

By differentiating with respect to χ and equating to zero, it can be shown that the total volume is a minimum when

$$\chi = \frac{2(1+\beta)}{2-\beta} k_1 k_2 \quad . \quad . \quad . \quad . \quad . \quad (26)$$

It will be seen that the optimum value of χ depends on k_1 and k_2, which are approximately constant for all chokes, and on β, which is determined by the core material and the value of $\hat{B}_\Delta$. As β does not vary much with $\hat{B}_\Delta$, provided the latter is small, and as the effect on the volume of the choke of slight deviations from optimum in the value of χ is small, it can be concluded that, for any given core material, a single value of χ will give minimum volume for all chokes irrespective of current rating or inductance, provided that the performance is specified in the manner assumed in developing the design method.

Considering now condition (*b*), it can readily be shown that if the ratio of the density of the conductor material to the density of the core material is y, the optimum value of χ will be y times the value of χ determined for minimum volume. For copper and for silicon-iron materials, the value of y is about 1·2; the effect on weight or volume of a change of χ in this ratio will be so small that, from a practical point of view, the conditions of designing for minimum volume and minimum weight may be regarded as identical.

From the dimensions of a large number of chokes of which a few examples are given in Table 14, average values of k_1 and k_2 are 0·007 and 6·5 respectively; for copper (hot) $\rho_w = 1{\cdot}9$ microhm-cm, and hence

$$K = \rho_w \frac{k_2}{k_1} = 1{\cdot}76 \times 10^{-3}$$

For 4% silicon-iron, the most commonly used core material, and for values of $\hat{B}_\Delta$ not exceeding 10 gauss, average experimental values of α and β are 0·0010 and 0·6 respectively. These values give a value of χ for minimum volume of about 0·1, and for minimum weight about 0·12.

Taking $\chi = 0{\cdot}1$ and assuming the above values of other constants, we have, for 4% silicon-iron,

$$\text{Volume of core} = 4\,700 \frac{I_p^{1\cdot69} L_\Delta^{1\cdot30}}{V_p^{0\cdot91}}\ \text{cm}^3 \quad . \quad . \quad (27)$$

and total volume of core plus winding conductor

$$= 6\,900 \frac{I_p^{1\cdot69} L_\Delta^{1\cdot30}}{V_p^{0\cdot91}}\ \text{cm}^3$$

$$= 0{\cdot}24 \frac{I_p^{1\cdot69} L_\Delta^{1\cdot30}}{V_p^{0\cdot91}}\ \text{ft}^3 \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The overall volume of the complete choke in case will usually be between 5 and 10 times the above volume.

With the same values of constants, the total weight of core and winding

$$= 120 \frac{I_p^{1\cdot69} L_\Delta^{1\cdot30}}{V_p^{0\cdot91}}\ \text{lb} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and

$$l = 78 \frac{I_p^{0\cdot56} L_\Delta^{0\cdot43}}{V_p^{0\cdot30}}\ \text{cm} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

From this eqn. and eqns. (16), (17), (18) and (25), all the design data for the choke can be obtained.

Fig. 7 gives, for Stalloy, a chart by means of which the weight or volume of the core and winding of a smoothing choke for which $V_p = 1$ volt can be directly determined. For a different value of V_p, the weight or volume from the chart should be multiplied by a factor as follows:

V_p	0·1	0·2	0·5	2	5	10
Factor	8·1	4·3	1·88	0·53	0·23	0·123

Similar charts can be prepared for other core materials using eqns. (28) and (29) and substituting the appropriate values for α and β.

(5.4.2) Design for a Given Maximum Temperature Rise.

In the majority of practical applications in which the authors have applied the method, it has been found that a design based on the permitted voltage drop will also give a tolerable temperature rise. In some instances, however, the latter consideration may be the limiting design factor.

The temperature rise can be approximately determined, if the power dissipated per unit area of winding surface is known, a value of 0·1 watt/cm² giving a rise of about 40° C.

The surface area of the winding is approximately proportional to l, the magnetic length of the core and to l_T the mean length of turn. If l_T is assumed, as in Section 5.4.1, to be proportional to $\sqrt{A}$, we can write

$$\text{Surface area} = k_3 l \sqrt{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

when k_3 is a constant. From data derived from a large number of chokes the average value of k_3 for shell-type chokes with a single winding is about 1·8 and for core-type chokes with two windings about 3·6. The latter construction would normally be used when temperature rise is likely to be an important factor. The power dissipated in the winding is $V_p I_p$, and hence if P is the power/cm²,

$$P = \frac{V_p I_p}{k_3 l \sqrt{A}} = \frac{V_p I_p}{k_3 \chi l^2} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The value of P for a choke designed by the method of Section 5.4.1 can be found by inserting the value of l given by eqn. (22). Making this substitution, and assuming constant values of α, β, χ, K and k_3, we have

$$P \propto \frac{V_p^{(8-\beta)/(4+\beta)}}{I_p^{\beta/(4+\beta)} L_\Delta^{4/(4+\beta)}} \quad . \quad . \quad . \quad . \quad (33)$$

Since, for the materials considered, β lies between 0·5 and 0·75, the index of I_p is very small. High values of P will therefore occur in chokes with large voltage drop or small inductance. In economically designed smoothing filters of the type considered, the range of values of V_p is small, and low values of L_Δ occur in filters for high values of I_p only. Hence it follows that, for

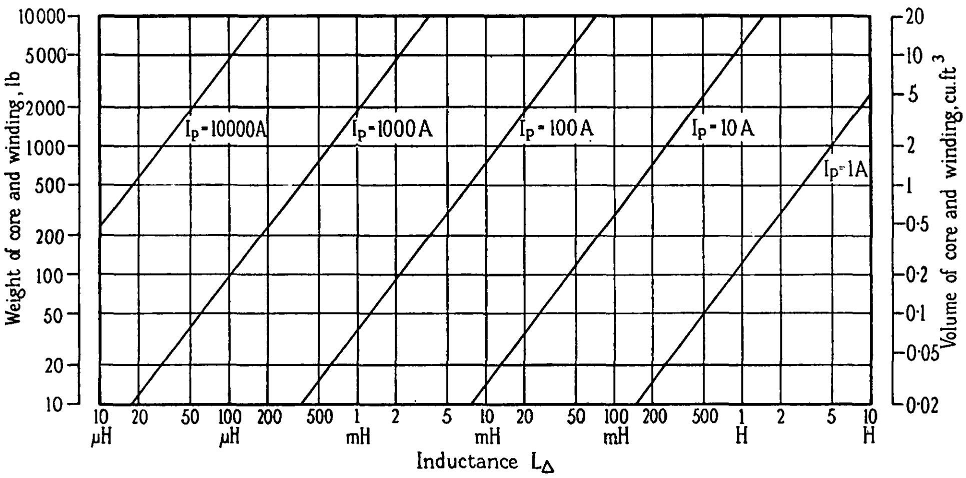

Fig. 7.—Chart to determine weight and volume of core and winding of a Stalloy-cored choke. $V_p = 1$ volt.

the particular application considered, temperature rise is a serious factor only in very heavy current chokes.

If the same values of constants are assumed as in eqns. (27) to (30), i.e. $\chi = 0 \cdot 1$, $\alpha = 0 \cdot 0010$ and $\beta = 0 \cdot 6$, the expression for P for a choke designed as in Section 5.4.1 becomes

$$P = 4 \cdot 6 \times 10^{-4} \frac{V_p^{1 \cdot 6}}{I_p^{0 \cdot 13} L_\Delta^{0 \cdot 87}} \quad . \quad . \quad . \quad . \quad (34)$$

for core-type two-winding chokes. For shell-type, single-winding chokes, the value of P is twice that given by eqn. (34).

If a limiting value for P of $0 \cdot 1$ watt/cm^2 is assumed, it can be shown from eqn. (34) that this value is likely to be exceeded in a choke whose inductance is less than about 1 mH when $V_p = 1$ volt and less than about 100 μH when $V_p = 0 \cdot 25$ volt.

If, in any instance, the value of P calculated in the above manner is too high, a re-design by the following method may be carried out. This method assumes a particular value of P, V_p being unknown, and as before, it is required to determine the design parameters l, N, etc., and the optimum value of χ.

From eqns. (21) and (30) we have

$$N = \frac{1}{I_p}\left(\frac{Pk_3 l^3}{K}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

N, $\nu'_{\Delta_{min}}$, and H_p can be eliminated from eqns. (11), (14), (20), and (35), and we obtain

$$l = \left[\frac{\alpha^2 L_\Delta^2 K^{2-\beta} I_p^4 \times 10^{16}}{(0 \cdot 4\pi)^{2-2\beta} \chi^4 k_3^{2-\beta} P^{2-\beta}}\right]^{\frac{1}{8-\beta}} \quad . \quad . \quad . \quad (36)$$

Whence core volume

$$= \chi^2 l^3 = \left[\frac{\chi^{4-2\beta} \alpha^6 L_\Delta K^{3(2-\beta)} I^{\;2} \times 10^{48}}{(0 \cdot 4\pi)^{6(1-\beta)} k_3^{3(2-\beta)} P^{3(2-\beta)}}\right]^{\frac{1}{8-\beta}} \quad . \quad (37)$$

The total volume of core plus winding is therefore proportional to

$$\left(\frac{k_1 k_2}{\chi} + 1\right)\chi^{\frac{4-2\beta}{8-\beta}}$$

By differentiating with respect to χ and equating to zero, it can be shown that the value of χ giving minimum total volume is

$$\chi = \frac{4 + \beta}{4 - 2\beta} k_1 k_2 \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Again, the value of χ for minimum weight will be y times the value for minimum volume. Taking as before a mean value of $\beta = 0 \cdot 6$ for 4% silicon-iron, and $k_1 = 0 \cdot 007$ and $k_2 = 6 \cdot 5$, the value of χ for minimum volume is $0 \cdot 075$ and for minimum weight $0 \cdot 09$. However, it is usual when a choke is designed on temperature rise to increase the radiating surface by spacing the turns, as it is generally economical to do so provided it does not lead to excessive voltage drop. The chief effect of this is to reduce k_1, so that the optimum values of χ will be rather less than those calculated above.

Assuming $\chi = 0 \cdot 075$, $k_3 = 3 \cdot 6$ (since a core-type construction would be adopted when temperature rise is serious) and, as before, $K = 1 \cdot 76 \times 10^{-3}$, $\alpha = 0 \cdot 0010$ and $\beta = 0 \cdot 6$ (for 4% silicon-iron), we have

$$\text{Volume of core} = 54 \frac{L_\Delta^{0 \cdot 81} I^{1 \cdot 62}}{P^{0 \cdot 57}} \text{ cm}^3 \quad . \quad . \quad . \quad (39)$$

and total volume of core plus winding

$$= 87 \frac{L_\Delta^{0 \cdot 81} I_p^{1 \cdot 62}}{P^{0 \cdot 57}} \text{ cm}^3$$

$$= 0 \cdot 003 \frac{L_\Delta^{0 \cdot 81} I_p^{1 \cdot 62}}{P^{0 \cdot 57}} \text{ ft}^3 \quad . \quad . \quad . \quad . \quad (40)$$

Again, with the same values of constants assumed, the total weight of core and winding

$$= 1 \cdot 66 \frac{L_\Delta^{0 \cdot 81} I_p^{1 \cdot 62}}{P^{0 \cdot 57}} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

and eqn. (30) becomes

$$l = 21 \cdot 3 \frac{L_\Delta^{0 \cdot 27} I_p^{0 \cdot 54}}{P^{0 \cdot 19}} \quad . \quad . \quad . \quad . \quad . \quad (42)$$

From this eqn. and eqns. (16), (17), (18) and (35), all the design data for the choke can be obtained.

(6) ACKNOWLEDGMENTS

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(8) APPENDICES

(8.1) Method of Measurement of Incremental Properties

Magnetization curves (relations between B_p and H_p) were measured by the ballistic galvanometer method. Fig. 8 shows the arrangement. The polarizing current, I_p, and the alternating magnetizing voltage V_Δ, were applied in series to the magnetizing winding and the internal impedance of both supplies was low in order to avoid distortion of the sinusoidal applied alternating voltage. No condenser was connected across the d.c. supply to reduce the impedance, as this would have caused the current to surge to a value greater than the steady value to which adjustment had been made.

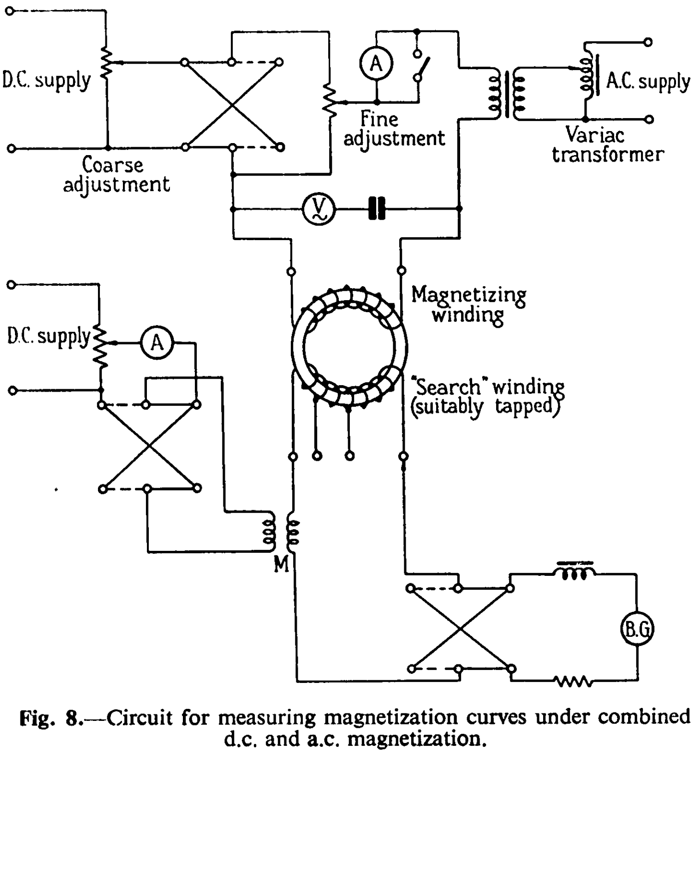

Fig. 8.—Circuit for measuring magnetization curves under combined d.c. and a.c. magnetization.

The specimen was first demagnetized from a value of $\hat{H}_\Delta$ of about 20 oersteds. This was conveniently carried out by means of a Variac transformer connected to the 50-c/s a.c. supply.

When the alternating voltage was applied for the purpose of making a magnetization curve measurement, it was applied slowly, as sudden application would have resulted in some residual magnetism. Furthermore, while the polarizing current was being reversed, the alternating voltage was maintained across the specimen in order to avoid a similar effect. The d.c. meter therefore had to be placed at a point in the circuit where the current changed direction and so a short-circuiting switch was provided, precautions being taken to avoid any change in conditions when the switch was closed. The use of this switch also avoided overloading the d.c. meter with alternating current. The alternating voltage was measured with a high-impedance instrument which was afterwards disconnected.

It was necessary to include in the galvanometer circuit a choke to limit the alternating current. In order to obtain conveniently sufficient inductance for this purpose an iron-cored choke was used. The current through it was maintained always in one direction so that no change in remanence occurred. (The galvanometer or fluxmeter would measure the total change in magnetic linkage in the circuit to which it is connected, and if such change in linkage is not confined to that occurring in the "search" coil of the specimen, serious inaccuracy would result.) This precaution was observed by reversing the connections to the search coil after each reversal of the polarizing current, so maintaining the galvanometer deflection always in one direction. Readings were taken only after several reversals of polarizing current and consequent deflections of the galvanometer.

The search winding was tapped so that a substantial galvanometer deflection was always obtained. The resistance of this winding was small so that there was no appreciable change in galvanometer sensitivity on changing taps.

Calibration of the galvanometer circuit was carried out using a standard air-cored mutual inductor, the secondary of which remained in circuit during all measurements in order to keep the resistance of the galvanometer circuit constant. During calibration the "search" winding on the specimen was short-circuited or removed from circuit to avoid any effect due to its magnetization by the galvanometer current.

Incremental permeability μ_Δ/θ was measured using the Owen bridge, which gives directly the inductance and effective series resistance of the magnetizing winding on the specimen. The permeability was calculated from these results, making allowance for the d.c. resistance of the winding. The arrangement of the Owen bridge is given in Fig. 9. The impedance of the a.c. supply was low, to avoid distortion of the sinusoidal voltage applied to the specimen, the blocking condenser being also of sufficient capacitance to achieve this. The meter used to measure the alternating voltage was of a high-impedance type but was disconnected while the bridge was actually being balanced.

The resistance of the bridge arm R_1 was as small as practicable to avoid distortion of the alternating flux density. This resistor was, of course, capable of carrying the polarizing current. It was convenient to use a 4-terminal resistor, the connections being as shown in Fig. 9.

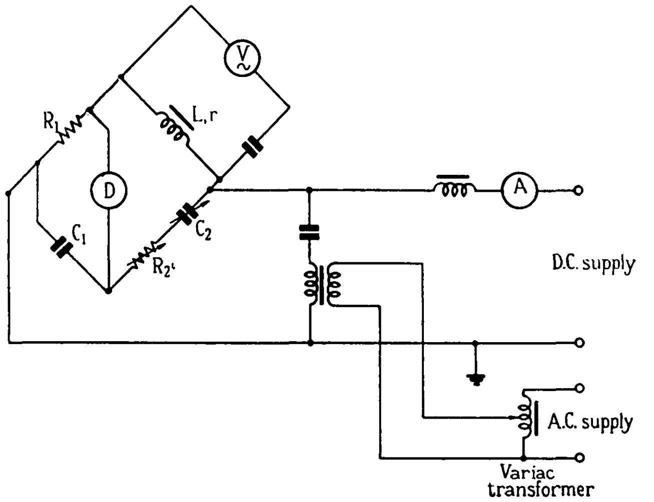

Fig. 9.—Circuit for measuring incremental permeability.

$$L = C_1 R_1 R_2 \qquad r = \frac{C_1 R_1}{C_2}$$

(8.2) Method of Determining the Approximate Value of $\hat{B}_\Delta$

In designing a choke, values of the constants α and β have to be assumed. For any particular material, however, α and β are dependent on the value of $\hat{B}_\Delta$, which cannot be determined until the design has been completed. It is therefore necessary to start by guessing the value of $\hat{B}_\Delta$, and to work out a design, then to calculate $\hat{B}_\Delta$ and, if this differs appreciably from the value assumed, to repeat the design with a closer estimate of $\hat{B}_\Delta$. In order to reduce the probability of requiring to use this method of successive approximation, the approximate value of $\hat{B}_\Delta$ may be determined without carrying out the complete design, as follows:

From eqns. (13), (14) and (22) it can be shown that

$$NA = \frac{\alpha}{(H_p')^{1-\beta}} L_\Delta I_p \times 10^{-8}$$

Hence from eqn. (10)

$$\hat{B}_\Delta = \frac{(H_p')^{1-\beta}}{\alpha} \frac{\bar{V}_\Delta}{\sqrt{(2)}\pi f L_\Delta I_p}$$

Now H_p' will almost invariably be between 20 and 200 oersteds, and $(1 - \beta)$ is of the order of $0 \cdot 25 - 0 \cdot 5$. Hence if a mean value of 60 is assumed for H_p', and a mean value of β for the material concerned, the value of $(H_p')^{1-\beta}$ will be correct within a factor of about $1 \cdot 5$. The possible error due to assuming a mean value of α is, in general, rather less. Thus the estimated value of $\hat{B}_\Delta$ should be correct within a factor of about 2. This accuracy is quite adequate to determine the more precise values of α and β to be assumed in the design.

(8.3) Results of Measurements on Nickel-Iron Alloys

A small number of measurements have been made on two nickel-iron alloys, namely Radiometal and Mumetal. The

Table 15

MUMETAL AND RADIOMETAL μ_p AND $\mu_\Delta/\bar{\theta}$ AS FUNCTIONS OF H_p

(a) Mumetal

	μ_p		μ_Δ, at 800 c/s				$\bar{\theta}$, at 800 c/s			
Thickness, mils	5	15	5		15		5		15	
H_p, oersteds	$\hat{B}_\Delta = 0$	$\hat{B}_\Delta = 0$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$
0	—	—	10 800	12 000	4 800	5 300	17°	23°	44°	48°
0·05	28 000	22 000	9 600	10 000	4 400	4 700	13°	16°	41°	44°
0·1	30 000	22 000	5 800	5 900	3 900	4 100	8°	10°	34°	34°
0·2	18 000	16 000	2 400	2 400	2 900	2 900	3°	4°	22°	22°
0·5	8 600	7 400	1 600	1 600	1 100	1 100	2°	2°	9°	9°
1·0	5 100	4 300	1 200	1 200	900	900	1°	2°	8°	8°
2·0	3 000	2 600	700	700	700	700	1°	2°	6°	6°
4·0	—	2 100	—	—	300	300	—	—	3°	3°

(b) Radiometal

	μ_p		μ_Δ, at 800 c/s				$\bar{\theta}$, at 800 c/s			
Thickness, mils	5	15	5		15		5		15	
H_p, oersteds	$\hat{B}_\Delta = 0$	$\hat{B}_\Delta = 0$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$	$\hat{B}_\Delta = 10$	$\hat{B}_\Delta = 100$
0	—	—	1 980	2 150	1 670	1 750	3°	6°	15°	16°
0·1	2 500	2 000	1 960	2 130	1 690	1 740	2°	6°	14°	16°
0·2	3 500	3 500	1 900	2 050	1 680	1 720	2°	6°	14°	15°
0·5	5 600	7 000	1 700	1 770	1 530	1 540	2°	5°	12°	13°
1·0	5 500	6 500	1 300	1 350	1 180	1 180	2°	3°	10°	11°
2·0	3 900	4 300	860	900	770	770	1°	2°	6°	7°
4·0	2 400	2 600	500	520	490	490	1°	1°	3°	4°
8·0	—	1 500	—	—	290	290	—	—	3°	3°

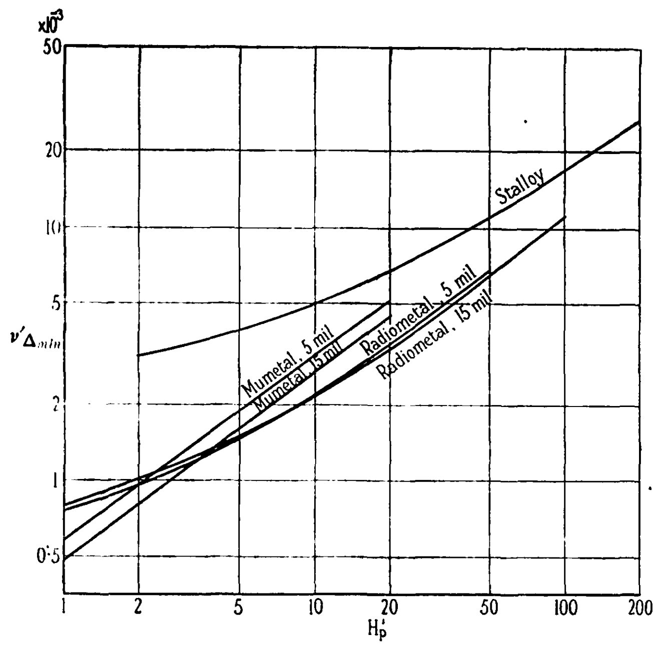

Fig. 10.—$\nu'_{\Delta_{min}}$ as a function of H'_p.

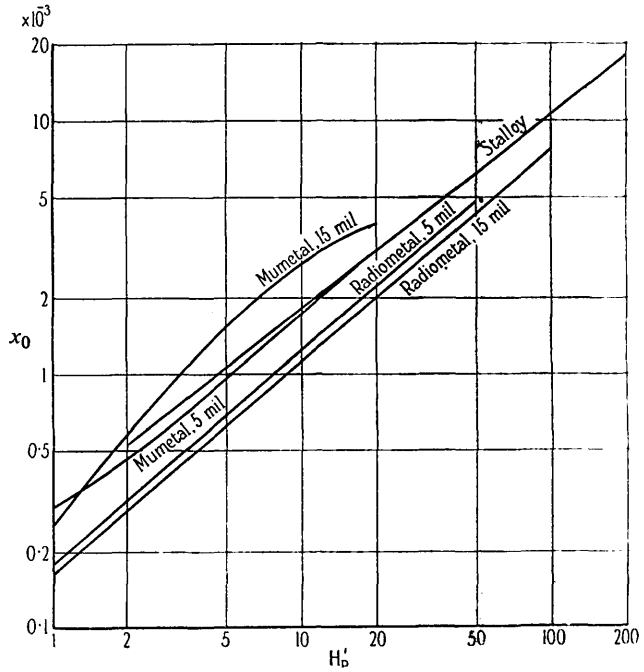

Fig. 11.—x_0 as a function of H'_p.

samples consisted of only two sets of laminations, one 5 mils thick and the other 15 mils thick, of each material. The samples were random and the results do not therefore necessarily represent the average performance of the material. They are included merely for rough comparison. The laminations were designed for transformer cores and when stacked each set gave a cross-section of $\frac{1}{2} \times \frac{1}{2}$ in. The cross-section of flux path was uniform except for the corners and only one butt joint occurred in the flux path of each lamination, the joints being staggered in alternate laminations. The windings were on a transformer bobbin mounted on the centre limb.

Measurements were made of μ_p for $B_\Delta = 0$ and μ_Δ for $\hat{B}_\Delta =$ 10 and 100 gauss over a range of H_p from 0 to 4 oersteds for Mumetal, and 0 to 8 oersteds for Radiometal. The results are given in Table 15.

$\nu'_{\Delta_{min}}$ and x_0 have been calculated from the results in Table 15 and plotted against H'_p on Figs. 10 and 11.

In the consideration of silicon-iron alloys in the main body of the paper, the relationship between $\nu'_{\Delta_{min}}$ and H'_p was assumed linear over a range of 10 : 1 in the value of H'_p. It will be seen that over a similar range the corresponding relationships for Mumetal and Radiometal are approximately linear, and that therefore the expression $\nu'_{\Delta_{min}} = \alpha(H'_p)^\beta$ could be assumed, provided values of α and β appropriate to the range of H'_p were adopted.

[The discussion on the above paper will be found on page 229.]